FEM BASED MODELLING AND ROBUST CONTROL OF TEMPERATURE FIELDS IN CASTING MOULD

Cyril BELAVÝ 1 – Marcel VLČEK – Filip VITÁLOŠ – Boris BARBOLYAS

Abstract: In the paper, modelling and robust control of temperature fields of the casting mould in the benchmark casting plant is presented. Temperature fields are supposed as distributed parameter systems with dynamic described by partial differential equations. Based on finite element method modelling a finite-dimensional approximation models are created. For the control synthesis with internal model control structure in the time domain, an uncertainty of the controlled system is considered and robust controllers are designed.

Abstrakt: V príspevku je prezentované modelovanie a robustné riadenie teplotných polí v zlievarenskej forme experimentálneho zlievarenského zariadenia. Teplotné polia sú uvažované ako systémy s rozloženými parametrami s dynamikou popísanou parciálnymi diferenciálnymi rovnicami. Na základe modelovania metódou konečných prvkov sú vytvorené konečno rozmerné aproximáčné modely. Pre účely syntézy riadenia so štruktúrou obvodu s vnútorným modelom v časovej oblasti je uvažovaná neurčitost riadeného systému a navrhnuté sú robustné regulátory.

Key words: casting mould, uncertainty, robust control, temperature fields

Kľúčové slová: zlievarenská forma, neurčitosť, robustné riadenie teplotné polia

INTRODUCTION

Most of the dynamical systems analysed in engineering practice have the dynamics, which depends on both position and time. Such systems are classified as distributed parameter systems (DPS). The casting technology is a typical case of the DPS. There in order to obtain the desired structure during solidification, the process requires a specific temperature field of the mould, which is defined on complex-shape 3D definition domain. Modelling, simulation and evaluation of real-time experiments in this area is now widely accepted as an important tool in product design and process development to improve productivity and casting quality.

For analysis of the casting process dynamics as DPS, especially temperature fields in the casting mould and control synthesis purposes, the benchmark casting plant with steel mould of complex-shape was designed. Temperature fields of the mould for casting processes in this benchmark were also modelled and studied by means of finite elements method (FEM) in the software environment COMSOL Multiphysics. Based on this FEM modelling, numerical models for DPS control synthesis purpose in the form of a lumped-input/distributed-output system (LDS) were created.

Next, uncertainty analysis of the models both in time and space domain was performed and robust control synthesis with internal model control (IMC) structure has been designed. Simulation of the robust control of temperature fields in the casting mould and afterwards real-time experiment in accordance to casting technology requirements was carried out using the Distributed Parameter Systems Blockset for MATLAB & Simulink, a third-party software product of The MathWorks, developed at the Institute of Automation, Measurement and Applied Informatics, Faculty of Mechanical Engineering STU Bratislava.

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1 LDS REPRESENTATION OF DPS

In general, DPS are systems whose state or output variables, \(X(x,y,z,t)\)/\(Y(x,y,z,t)\) are distributed variables or fields of variables, where \((x,y,z)\) are spatial coordinates in 3D. The time-space coupled nature of the DPS is usually mathematically described by partial differential equations (PDE) as infinite-dimensional systems [1], [2].

However, from point of view of implementation of DPS control in technological practice, where a finite number of sensors and actuators for practical sensing and control is at disposal, such infinite-dimensional systems need to be approximated by finite-dimensional systems. There are many dimension reduction methods, which can be used to solve this problem. Continuous and approximation theories aimed to control of parabolic systems presents monograph [3]. Methodical approach from the view of time-space separation with model reduction is presented in [4]. Variety of transfer functions for systems described by PDE are illustrated by means of several examples in [5]. Well-known reduction methods based on FEM, or finite difference method (FDM), spectral method require an accurate nominal PDE model and usually lead to a high-order model, which requires unpractical high-order controller.

In the input-output relation, PDE defined distributed-input/distributed-output systems (DDS) between distributed input \(U(x,y,z,t)\) and distributed output variables \(Y(x,y,z,t)\), at initial and boundary conditions given. DPS very frequently are found in various technical and non-technical branches in the form of LDS, (Fig. 1), [6], [7].

![Fig. 1 Lumped-input/distributed-output system representation of distributed parameter systems](image)

1.1 Dynamic of LDS

Output of the linear LDS in time domain, or in s-domain is in the form:

\[
Y(\bar{x},t) = \sum_{i=1}^{n} Y_i(\bar{x},t) = \sum_{i=1}^{n} G_i(\bar{x},t) \otimes U_i(t)
\]

(1)

\[
Y(\bar{x},s) = \sum_{i=1}^{n} Y_i(\bar{x},s) = \sum_{i=1}^{n} S_i(\bar{x},s) U_i(s)
\]

(2)

where \(\bar{x} = (x,y,z)\) is position vector in 3D, \(U_i(t)\) - lumped input variable, \(G_i(\bar{x},t)\) - i-th distributed impulse response, \(\otimes\) - denotes convolution product, \(Y_i(\bar{x},t)\) - system response to the i-th input, \(S_i(\bar{x},s)\) - i-th transfer function. When \(U_i(t)\) is a unit-step (Heaviside) function, \(Y_i(\bar{x},t)\) is in the form of distributed transient response function \(\mathcal{H}_i(\bar{x},t)\).

For a discrete-time system considering zero-order hold (ZOH) units, the overall distributed output variable of LDS and ZOH can be expressed as convolution sum \(\otimes\) in the form:

\[
Y(\bar{x},k) = \sum_{i=1}^{n} Y_i(\bar{x},k) = \sum_{i=1}^{n} G H_i(\bar{x},k) \otimes U_i(k)
\]

(3)
For points \( \mathbf{x}_i = (x_i, y_i, z_i) \) located in surroundings of lumped input variables, \( U_i(t) \), where partial distributed transient responses \( \mathcal{H}_i(\mathbf{x}_i, t) \) attain maximal amplitudes, partial distributed output variables are obtained in time-domain and next either continuous \( \{S_i(\mathbf{x}_i, s)\}_i \), or discrete transfer functions \( \{SH_i(\mathbf{x}_i, z)\}_i \) with sampling period \( T \) are identified.

\[
\{Y_i(\mathbf{x}_i, t) = \mathcal{G}_i(\mathbf{x}_i, t) \otimes U_i(t)\}_{i=1,n} \quad \rightarrow \quad \{Y_i(\mathbf{x}_i, s) = S_i(\mathbf{x}_i, s)U_i(s)\}_{i=1,n}
\]

\[
\{Y_i(\mathbf{x}_i, k) = \mathcal{G}H_i(\mathbf{x}_i, k) \otimes U_i(k)\}_{i=1,n} \quad \rightarrow \quad \{Y_i(\mathbf{x}_i, z) = SH_i(\mathbf{x}_i, z)U_i(z)\}_{i=1,n}
\]

For the space dependency and in steady-state we can define reduced transient step responses between \( i \)-th input variable at point \( \mathbf{x}_i = (x_i, y_i, z_i) \) and corresponding partial particular distributed output variable in steady-state:

\[
\begin{align*}
\mathcal{H}HR_i(\mathbf{x}_i, \infty) &= \mathcal{H}H_i(\mathbf{x}_i, \infty) \\
\mathcal{H}H_i(\mathbf{x}_i, \infty) &= \mathcal{H}H_i(\mathbf{x}_i, \infty)
\end{align*}
\]

Dynamics of LDS is decomposed to time and space components. In the time dependency, there are transfer functions either in the form \( \{S_i(\mathbf{x}_i, s)\}_i \), or \( \{SH_i(\mathbf{x}_i, z)\}_i \) with sampling period \( T \). In the space direction there are \( \{\mathcal{H}HR_i(\mathbf{x}_i, \infty)\}_i \).

1.2 Feedback control loop based on LDS

Decomposition of dynamics enables also to decompose the control synthesis to time and space control tasks in the DPS feedback control loop, (Fig. 2).

Fig. 2 Distributed parameter feedback control loop: HLDS - LDS with zero-order holds \( \{H_i\}_i \) on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, \( K \) - time/space sampling, \( V(\mathbf{x}, t) \) - disturbance variable, \( Y(\mathbf{x}, t) \) - distributed controlled variable, \( Y(\mathbf{x}, k) \) - sampled distributed controlled variable, \( Y_i(k) \) - approximation parameters of controlled variable, \( W(\mathbf{x}, k) \) - reference variable, \( \bar{W}_i(k) \) - approximation parameters of reference variable, \( E_i(k) \) - control errors, \( C_i(z) \) - controllers, \( U_i(k) \) - lumped control variables

Let us consider a step change of distributed parameter control variable \( W(\mathbf{x}, k) = W(\mathbf{x}, \infty) \) and \( V(\mathbf{x}, t) = 0 \). The goal of the control synthesis is to generate a sequence of control inputs \( \bar{U}(k) \) in such manner, that in the steady-state, for \( k \to \infty \), the control error \( E(\mathbf{x}, k) \) will approach its minimal value \( \|E(\mathbf{x}, \infty)\| \) in the quadratic norm:
min \|E(\bar{x},\infty)\| = \min \|W(\bar{x},\infty) - Y(\bar{x},\infty)\| = \|\bar{E}(\bar{x},\infty)\| \quad (7)

First, in the SS blocks, the approximation both of sampled distributed controlled variable \(Y(\bar{x},k)\) and reference variable \(W(\bar{x},\infty)\), on the set of reduced steady-state distributed step responses \(\{\mathcal{HR}_i(\bar{x},\infty)\}\), are solved in following form:

\[
\min_{Y_i} \left\| Y(\bar{x},k) - \sum_{i=1}^{n} Y_i(\bar{x},k) \mathcal{HR}_i(\bar{x},\infty) \right\| = \left\| Y(\bar{x},k) - \sum_{i=1}^{n} \bar{Y}_i(k) \mathcal{HR}_i(\bar{x},\infty) \right\| \quad (8)
\]

\[
\min_{W_i} \left\| W(\bar{x},\infty) - \sum_{i=1}^{n} W_i(\bar{x},\infty) \mathcal{HR}_i(\bar{x},\infty) \right\| = \left\| W(\bar{x},\infty) - \sum_{i=1}^{n} \bar{W}_i \mathcal{HR}_i(\bar{x},\infty) \right\| \quad (9)
\]

Basis functions (6) form a finite-dimensional subspace of approximation functions in the strictly convex normed linear space of distributed parameter quantities with quadratic norm, where the approximation problem is solved. From approximation theory involves, that solution of the approximation problems (8), (9) is guaranteed as a unique the best approximation in the form \(\sum_{i=1}^{n} \bar{Y}_i(k) \mathcal{HR}_i(\bar{x},\infty)\) with the vector of optimal approximation parameters \(\{\bar{Y}_i(k)\}\) in task (8) and the best approximation in the form \(\sum_{i=1}^{n} \bar{W}_i \mathcal{HR}_i(\bar{x},\infty)\) with the vector of optimal approximation parameters \(\{\bar{W}_i\}\) for approximation task (9).

Let us formulate a DPS control problem for the distributed reference variable \(W(\bar{x},\infty)\). When \(W(\bar{x},k)\) is assumed, the space control synthesis is performed in each time step \(k\), which gives \(\{\bar{W}_i(k)\}\) parameters. Next, based on the solution of approximation problem, the vector of control error is created:

\[
\bar{E}(k) = \{E_i(k)\} = \{\bar{W}_i - \bar{Y}_i(k)\} \quad (10)
\]

The control errors vector \(\bar{E}(k) = \{E_i(k)\}\) enters into the block TS, where the vector of control quantities, \(\bar{U}(k) = \{U_i(k)\}\), is generated by controllers \(\{C_i(z)\}\) in single-parameter control loops. During the control process, for \(k \to \infty\) the control task (7) is accomplished.

2 ROBUST CONTROL SYSTEM

In general, a mathematical model for the plant dynamics is the basis for analysis and design of control systems. Also for LDS representation of DPS lumped and distributed models are used. However, in practice is obvious that although no model is able to represent the process perfectly, some of them will do so with greater accuracy than others.

The theory of robust control represents one of the possible approaches to the control system design in the presence of uncertainty. The goal of robust system design is to retain a good quality of system performance in spite of model inaccuracies and changes. For the design techniques, the following requirements are supposed to be fulfilled: formulation of nominal plant model, different plant uncertainty models and requirements for both, robust stability and performance.
2.1 Sources of uncertainties in the LDS structure

LDS representation of DPS means decomposition of dynamics to space and time components. Uncertainties may occur in both, time and space components. In DPS control system, (Fig. 2) single-input, single-output control loops in the block TS are tuned as closed feedback control loops using known methods. In these loops, as models of the controlled system, transfer functions \( \{SH_i(\bar{x}, z)\} \) and/or \( \{S_i(\bar{x}, s)\} \) in the s-domain are used. These transfer functions describe the dynamics between sequences \( \{U_i(k)\} \) and \( \{Y_i(\bar{x}, k)\} \). In this case, sources of uncertainties are given mainly by procedure of dynamics modelling and possible change of parameters in models (4), (5) and solution of approximation problems, where lumped variables are obtained (8), (9).

In order to treat uncertainties, it will be assumed in this paper that the dynamic behavior of a plant is described not by a single linear time invariant model, but by a family of linear time invariant models, \( \Psi_i \). This family \( \Psi_i \) in the frequency domain, e.g. for models \( \{S_i(\bar{x}, s)\} \), takes the form:

\[
\Psi_i = \left\{ S_i : \left| S_i(\bar{x}_i, j\omega) - \tilde{S}_i(\bar{x}_i, j\omega) \right| \leq \bar{L}_ui(\omega) \right\}
\]

where \( \tilde{S}_i(\bar{x}_i, j\omega) \) is the nominal plant. Any member of the family \( \Psi_i \) fulfills the conditions:

\[
S_i(\bar{x}_i, j\omega) = \tilde{S}_i(\bar{x}_i, j\omega) + L_{ui}(j\omega)
\]

\[
|L_{ui}(j\omega)| \leq \bar{L}_{ui}(\omega), \ \forall \ S_i \in \Psi_i
\]

where \( L_{ui}(j\omega) \) is an additive uncertainty and \( \bar{L}_{ui}(\omega) \) is the bound of additive uncertainty. If we wish to work with multiplicative uncertainties, we define the relations:

\[
L_{mi}(j\omega) = \frac{L_{ui}(j\omega)}{S_i(\bar{x}_i, j\omega)}, \quad \bar{L}_{mi}(\omega) = \frac{\bar{L}_{ui}(\omega)}{|S_i(\bar{x}_i, j\omega)|}
\]

2.2 Design of IMC robust controllers

A robust control system for LDS can be designed, for example using the IMC structure [8], [9], [10]. This well-known structure is incorporated into the TS block of the DPS feedback control system, (Fig. 3).
$H_2$ optimal IMC controllers $\{Q_i(z)\}_{i}$ for inputs $\{\tilde{W}_i(k)\}_{i}$ in the form unit-step function $\gamma(z) = \frac{z}{z-1}$ are obtained from solution of the following minimization problem:

$$\min_{Q_i(z)} \|e_i(z)\|_2 = \min_{Q_i(z)} \|1 - SH_i(\bar{x}_i, z)Q_i(z)\|_2 \gamma_i(z)$$  \hspace{1cm} (15)$$

subject to the constraint $Q_i(z)$ to be stable and causal.

First, factorize the nominal stable transfer function $SH_i(x_i, z)$:

$$SH_i(\bar{x}_i, z) = SH_{N_i}(\bar{x}_i, z) SH_{M_i}(\bar{x}_i, z)$$  \hspace{1cm} (16)$$

where $SH_{N_i}(\bar{x}_i, z)$ includes positive zeros or time-delays of the transfer function $SH_i(x_i, z)$.

After this, optimal IMC controller $Q_i(z)$ is given by:

$$Q_i(z) = SH_{M_i}(\bar{x}_i, z)^{-1}$$  \hspace{1cm} (17)$$

Finally, controller $Q_i(z) Q_j(z)$ is augmented by low-pass filter $F_i(z)$ with parameter $0 < \alpha_i < 1$:

$$F_i(z) = \frac{1-\alpha_i}{z-\alpha_i}$$  \hspace{1cm} (18)$$

Resulting IMC controller with filter $Q_{fi}(z)$ is in following form:

$$Q_{fi}(z) = Q_i(z) F_i(z) = Q_i(z) \frac{(1-\alpha_i)}{z-\alpha_i}$$  \hspace{1cm} (19)$$

Parameter of the filter $\alpha_i$ is the only one tuning parameter to be selected by the user to achieve the appropriate compromise between performance and robustness and to keep the action of the manipulated variable within bounds. It must be chosen with respect to both, robust stability and robust performance condition for $0 \leq \omega \leq \pi / T$:

$$|F_i(e^{j\omega})| < \left[ |S_i(\bar{x}_i, j\omega)Q_i(e^{j\omega})|L_m(\omega) \right]^{-1}$$  \hspace{1cm} (20)$$

$$\left| Q_i(j\omega) |\bar{L}_m(\omega)|+1-S_i(\bar{x}_i, j\omega)Q_i(j\omega)|G_{u1}(\omega) \right| < 1$$  \hspace{1cm} (21)$$

where $G_{u1}(\omega)$ is a weighting function.

For $SH_i(x_i, z) = SH_{M_i}(\bar{x}_i, z)$ and low-pass filter (18), robust controller $C_j(z)$ in equivalent classical feedback control loop takes the form:

$$C_j(z) = \frac{SH_{M_i}(\bar{x}_i, z)^{-1} F_j(z)}{1 - SH_i(\bar{x}_i, z) SH_{M_i}(\bar{x}_i, z)^{-1} F_j(z)} = \frac{1}{SH_{M_i}(\bar{x}_i, z)} \frac{F_i(z)}{1-F_i(z)}$$  \hspace{1cm} (22)$$
3 MODELLING OF TEMPERATURE FIELD OF THE CASTING MOULD

The casting mould is one of the key components of a casting. It is well known, that the quality of the castings is affected strongly by the surface quality and the distribution of temperature in the mould, which has both time and space dependence. For study of the physical phenomena occurring during the casting solidification, from a DPS control point of view, control system development as well as mathematical model validation, a benchmark of the casting processes with two-part steel mould of a complex-shape was designed, (Fig. 4).

![Fig. 4 Benchmark casting plant and bottom side of the steel casting mould](image)

Inside of the casting mould are built-in 26 electric heating elements in the cylinder shape, each with maximal heating power 400 W. Heating elements are grouped to 5 zones and their heating power is actuated by the input voltage range of (0–10) V. In the body of the mould is also placed 7 water-cooled copper chills and 11 thermocouples, see Fig. 5. Location of these elements has been carefully designed in order to have the possibility of preheating the mould achieving desired temperature profile.

Nowadays a number of software products, environments and virtual try-out spaces for numerical dynamical analysis of machines and processes are at disposal, practically in all engineering disciplines. For modelling of temperature fields as DPS based on FEM, wide possibilities are offered by COMSOL Multiphysics Environment with Heat Transfer Module.

Distribution of temperatures $T(\vec{x},t)$ in the casting mould over the definition domain $\Omega \in E_3$ for lumped input variables $\{U_i(\vec{x},t)\}_{i=1,5}$ in the form of heat sources (W/m$^3$) is modelled by PDE of parabolic type with initial condition $T_{init}$ and boundary conditions (BC) for heat flux:

$$\frac{\partial T(\vec{x},t)}{\partial t} - a \nabla^2 T(\vec{x},t) = \sum_{i=1}^{5} U_i(\vec{x},t)$$

$$-n(-\hat{A}\nabla T) = h(T_{ext} - T)$$

(23)

$$T(\vec{x},0) = T_{init}$$

where $a = \lambda/(\rho C_p)$ is temperature conductivity (m$^2$.s$^{-1}$). Parameters of PDE and BC are in (Tab. 1) and values of heat sources in each zone are in (Tab. 2).
Tab. 1 Parameters of PDE for Iron

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>76.2</td>
<td>W/(m.K)</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>ρ</td>
<td>7870</td>
<td>kg/m³</td>
<td>Density</td>
</tr>
<tr>
<td>C_p</td>
<td>440</td>
<td>J/(kg.K)</td>
<td>Heat capacity</td>
</tr>
<tr>
<td>h</td>
<td>10</td>
<td>W/(m².K)</td>
<td>Heat transfer coeff</td>
</tr>
</tbody>
</table>

Tab. 2 Heat sources in zones of casting mould

<table>
<thead>
<tr>
<th>Zone #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat sources $Q_i$ [W/m³]</td>
<td>2.91026·10^7</td>
<td>4.36539·10^7</td>
<td>2.91026·10^7</td>
<td>2.91026·10^7</td>
<td>5.82052·10^7</td>
</tr>
</tbody>
</table>

For lumped input variables, $\{U_i\}_{i=1,5}$, in the form of step functions, which operate separately on sub-domains $\{\Omega_i\}_{i=1,5}$, all necessary lumped and distributed responses were computed by FEM. Results e.g. for actuating with heat source $U_4 = 2.91·10^7$ W/m³ in 4th zone are depicted in (Fig. 5).

![Fig. 5 Distributed transient responses in steady-state $\mathcal{H}_4(\bar{x},\infty)$ from input $U_4$ and transient responses at positions where thermocouples are placed](image)

Next, transient responses with maximal amplitude (e.g. for actuating in 4th zone it is position 4) were used for identification of transfer functions $\{S_i(\bar{x},s)\}_{i=1,5}$ in the form:

$$S_i(\bar{x},s) = \frac{K(Tz_s + 1)}{(Tp_1s + 1)(Tp_2s + 1)}$$ (24)

4 UNCERTAINTY ANALYSIS OF MODELS AND ROBUST CONTROL

In the block SS of the DPS feedback control system (Fig.3), approximations $\{\bar{Y}_i(k)\}_i$ from task (8) are obtained. Dynamics of these lumped variables is different like dynamics of lumped variables, given by transfer functions $\{SH_i(\bar{x},z)\}_i$. Uncertainty due to approximation in SS was analysed with DPS Blockset and Simulink blocks. Uncertainty region caused by this reason is possible to cover by appropriate variability of parameters $Tp_1, Tp_2$, see (Fig. 6).
Fig. 6 Uncertainty region in Zone # 4 caused by the approximation in the SS and its cover by ranges of parameters $T_p_1$, $T_p_2$ of the transfer function $S_i(\bar{x}, s)$

Finally, IMC based robust controllers (19) were designed. In the MATLAB & Simulink environment by means of the DPS Blockset, DPS IMC robust control scheme was arranged (Fig.7). There inside of the block DPS IMC Control synthesis both space and time control synthesis is solved. In the blocks DPS Space Synthesis, approximation tasks (8), (9) are solved and in the block Controllers based on IMC, 5 robust controllers are used. Parameters of filters $\{\alpha_i\}_{i=1,5}$ were optimized in the block Output Constraint according to criterion function (25) in order to assure nearly aperiodic course of the quadratic norm of the control error.

$$J = \min_{\alpha} \sum_{k=0}^{N} \left\| W(\bar{x},k) - Y(\bar{x},k) \right\|$$  \hspace{1cm} (25)

Fig. 7 DPS IMC robust control block scheme in MATLAB-Simulink and DPS Blockset

Optimized robust controllers were then implemented to the control scheme for the real-time control of temperature fields of the casting mould in preheating process. Robust control according scheme on (Fig. 3) was realized for the distributed reference variable - temperature given in 11 positions, where thermocouples are embedded. Results of the real-time robust control process are on (Fig. 8). The performance of the control both in the time and space domain is given by the quadratic norm of the distributed control error.
Fig. 8 Robust control of temperature field of the casting mould: distributed reference variables, controlled variables at given position of the mould, quadratic norm of distributed control error and control variables $U_i(k)$ in Zones 1-5

5 CONCLUSIONS

Nowadays is actual both to formulate and solve tasks of control in various engineering branches, including casting area, by means of methods and tools of DPS. Modelling and study of temperature fields of the mould in the casting process as DPS is the key solution to reduce manufacturing costs, shorten lead times for mould developments and improve the casting process quality. Methodical approach presented in this paper demonstrates possibilities, how to exploit of distributed dynamical characteristics, obtained by numerical FEM modelling and analysis of systems on complex definition domains. Created DPS/LDS models are useful not only for study of their dynamics, but also for design of various control synthesis methods both in time and space dependency with respect to uncertainty of models.

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